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## POSIDONIUS AND THE SOURCES OF PYTHAGOREAN ARITHMOLOGY<sup>1</sup>

BY FRANK EGLESTON ROBBINS

It has become the practice in recent years, especially in Germany, to regard Posidonius' *Commentary on the Timaeus* as the source of scientific pronouncements of various kinds in the ancient authors. Although this commentary was undoubtedly a most important and comprehensive work, it nevertheless seems somewhat strange, to mention at once the matter with which these pages are concerned, that the Greeks invariably went thither for information about Pythagorean number-theories at a time when the literary world was flooded with books, pseudonymous or otherwise, written by the Pythagoreans themselves.

Posidonius became recognized as the chief ancient authority upon the Pythagorean lore of numbers chiefly through the arguments of A. Schmekel,<sup>2</sup> who used as the basis of his theory Sextus Empiricus *Adv. math.* vii. 91 ff. Since that time his conclusion has

<sup>1</sup> "Arithmology" has been defined as "ce genre de remarques sur la formation, la valeur et l'importance des dix premiers nombres, où se mêlent la saine recherche scientifique et les fantaisies de la religion et de philosophie" by A. Delatte, "Études sur la littérature pythagoricienne," *Bibl. de l'École des Hautes Études*, fasc. 217, Paris, 1915, p. 139.

<sup>2</sup> *Die Philosophie der mittleren Stoa in ihrem geschichtlichen Zusammenhange dargestellt*, Berlin, 1892, pp. 403 ff.

generally been accepted as final.<sup>1</sup> More may be drawn from the study of Sextus, however, than Schmekel took account of.

In the first place, Schmekel used the passage designated almost to the exclusion of another in the same work, iv. 2 ff., which, since it agrees practically word for word with vii. 91 ff. throughout much of its length, is obviously ultimately derived from the same source. A proper diagnosis of the situation surely demands, first of all, that the two passages be examined side by side and their meaning analyzed;<sup>2</sup> hence one may infer whence each came and for what purpose it was written. If we do this, and take iv. 2 ff. first, we find that its outline is as follows: At the very beginning the general theme of the passage is stated, which is simply that the Pythagoreans ascribed great power to numbers in the government of the universe; this caused them to use the expression "all things are like to number," and in their well-known oath by Pythagoras to speak of the tetrakty as the "source that holds the roots of everlasting nature." The following sections develop the implications of this epithet and are therefore thoroughly consistent with the central theme of the passage; the numbers of the tetrakty, 1, 2, 3, 4, in their capacities as point, line, surface, and solid, respectively, represent all bodily natures, and, furthermore, soul; for since 1, 2, 3, and 4 contain the fundamental concords they control the harmony whereby the world is ordered and the soul constituted. This passage, then, is thoroughly consistent within itself, its main thesis is kept in view throughout and supported by argument, and its logical structure is admirable.

The other, vii. 91 ff., treats an entirely different theme, the criterion. It may be summarized thus: Anaxagoras declared that the criterion was *λόγος*, using the term in a general way; but the Pythagoreans restrict it to the mathematical *λόγος*, because, as Philolaus said, if it takes cognizance of the universe it must be akin

<sup>1</sup> Cf. G. Borghorst, *De Anatolii fontibus*, Berlin, 1905; G. Altmann, *De Posidonio Timaei Platonis commentatore*, Berlin, 1906; Gronau, *Poseidonios und die jüdisch-christliche Genesexegese*, Berlin, 1914, p. 197, n. 1.

<sup>2</sup> Zeller, *Philosophie der Griechen*, III, 1, p. 514, n. 5, concludes from such a comparison that the Pythagorean element in *Adv. math.* vii, 92 ff. is not from Posidonius. Against this view Schmekel argues, *op. cit.*, p. 407, n. 1. A proper comparative analysis of the two passages has not, however, been made.

to the universe, on the principle of the perception of like by like; Empedocles, who is quoted,<sup>1</sup> clearly states this principle. And as the light is perceived by the kindred sight, so Posidonius says in his *Commentary on the Timaeus*, and sound by the hearing, which is akin to air, so the universe ought to be apprehended by a *λόγος* of the same nature. But number is the source of the universe, wherefore the *λόγος* that is the criterion of all, not without its share in this, would be called number. It is to declare this that the Pythagoreans often say “all things are like to number” and use their accustomed oath. Its author was Pythagoras; the tetraktys is 10, or 1+2+3+4, which is the “source of everlasting nature,” first, because it contains the basic concords; second, because it contains point, line, surface, and solid; furthermore, because, all things being numerable, number is fundamental. This is the outline of the passage; I have more carefully reproduced the first portion in order to make clear the logical connection; but the last part, from the mention of the Pythagorean sayings, is that which is closest to iv. 2 ff., and no argument is needed to show that ultimately the two must have been derived from the same source.

Now it must be equally clear, it seems to the writer, that there are signs of disjointedness and of compilation in the second, but not in the first, passage. For in the former there is one theme only, and that sustained consistently throughout; in the latter we begin with the criterion and then abandon it for a detailed discussion of certain Pythagorean theories about number, the transition being of the most formal character. The last portion of vii. 91 ff. leaves the impression of being an argument, already existent and ready to hand in the work of some earlier author, awkwardly forced into a new context by the writer of the part about the criterion, and betraying its alien origin by the noticeable lapse in logical connection. This conclusion commends itself the more strongly because we know iv. 2 ff. also; having the latter, and seeing in it none of the marks of patching together, but on the contrary an admirable unity of construction, we must conclude that in iv. 2 ff. we have Sextus' quotation of certain material in its original form and used as it was

<sup>1</sup> Fragment 109, Diels; cited also by Aristotle *De anima* A 2 404 b 8; *Metaphysica* B 4 1000 b 5; and by Chalcidius in *Plat. Tim.* c. 50.

originally intended to be used, whereas in vii. 92 ff. he has employed a passage of another author who had himself cited this same material.

There is still further proof that the Pythagorean part of vii. 92 ff. existed in another form (i.e., that of iv. 2 ff.) before it was taken into its present context; not only that, but we may infer the original purpose of its composition. This is afforded by the discovery of a probably identical origin for *Adv. math.* iv. 2 ff. and two passages of Theon of Smyrna and Anatolius respectively.

Sextus Empiricus *Adv. math.* iv. 2 ff.:

καθόλου μὲν οὖν οἱ ἀπὸ τῶν μαθημάτων Πυθαγορικοὶ μεγάλην ἀπορέμονσι δύναμιν τοῖς ἀριθμοῖς ὡς τῆς τῶν δικαιωρῶν φύσεως κατ' αὐτοὺς διοικουμένης. Θεν καὶ ἀεὶ ποτε ἐπεφάνουν τὸ “ἀριθμῷ δὲ τε πάντι” ἐπεικεν, δύναντες οὖν μόνον τὸν ἀριθμὸν ἀλλὰ καὶ τὸν ὑποδείκνατα αὐτοῖς τὸντον Πυθαγόραν ὡς θεὸν διὰ τὴν ἐν ἀριθμητικῇ δύναμιν, λέγοντες “Οὐ μά τὸν ἀμετέρῳ φυχῇ παραδόντα τετρακτύν, Πηγὴν ἀενάου φύσεως βίζωματ’ ἔχουσαν.” τετρακτὺς δὲ προσηγορεύετο παρ’ αὐτοῖς ὁ ἐκ τῶν πρώτων δ’ ἀριθμῶν

Theon, p. 99, ll. 8-23:<sup>1</sup>

τοῦτων συστὰς κόσμος ἔσται

<τέλειος> ἡρμοσμένος κατὰ γεωμετρίαν καὶ ἀρμονίαν καὶ ἀριθμόν, δυνάμει περιεληφθὼς πᾶσαν ἀριθμοῦ φύσιν πᾶν τε μέγεθος καὶ πᾶν σῶμα ἀπλοῦν τε καὶ σύνθετον, τέλειός τε, ἐπειδὴ τὰ πάντα μὲν τούτου μέρη, αὐτὸς δὲ οὐδενός. διὸ πρώτῳ τῷ εἰρημένῳ ὅρκῳ οἱ Πυθαγορικοὶ ἐλέγοντο<sup>3</sup> . . . καὶ ἀριθμῷ δὲ τε πάντι” ἐπεικε. καὶ τοῦτο εἴγαι τὸ σοφώτατον πάντα μὲν γάρ τὸν ἀριθμὸν εἰς δεκάδα ἡγαγοῦν, ἐπειδὴ ὑπὲρ δεκάδα οὐδεὶς ἔστιν ἀριθμός, ἐν τῇ αὐξῆσει πάλιν ἡμῶν

Anatolius, p. 29, 1-10:<sup>2</sup>

ἡ δεκάδος καὶ τῶν ἐντὸς αὐτῆς ἀριθμῶν φύσις μυρία μὲν περιέχεται καὶ ἐπιδεκνυσι κάλλη τοῖς δύναμέρως τῷ νῷ καθορᾶν τὰ τοιαῦτα δυναμένοις. δσα μὲν οὖν ἐὰν οἶδον τε ἡ λέξομεν ἐφ’ ἐκάστου τῶν ἀριθμῶν, τοσοῦτον δὲ προλέγομεν νῦν,

ὅτι οἱ Πυθαγορεῖοι τὸν πάντα ἀριθμὸν εἰς δέκα ἀνηγον, ὑπὲρ δέκα δὲ οὐδεὶς ἔτι ἀριθμός, ἐν πάσῃ αὐξῆσει πάλιν ἡμῶν ἐπι-

<sup>1</sup> *Expositio rerum mathematicarum ad legendum Platonem utilium*, ed. Hiller, Leipzig, 1878. The work is generally acknowledged to be a compilation. Schmeling, *op. cit.*, p. 410, believes this passage to come from Thrasyllos; so also Switalski, *Des Chalcidius Kommentar zu Platos Timaeus, Beiträge z. Gesch. d. Phil. d. Mittelalters*, III, 6, Münster, 1902, pp. 85-86; Borghorst, *op. cit.*, pp. 17 ff., assigns it to Moderatus, and Altmann, *op. cit.*, pp. 19 ff., to Adrastus.

<sup>2</sup> Ed. J. L. Heiberg, *Annales internationales d’ histoire, Congrès de Paris, 1900, 5<sup>η</sup> section, histoire des sciences*, Paris, 1901. The title is περὶ δεκάδος καὶ τῶν ἐντὸς αὐτῆς ἀριθμῶν. Many excerpts from it appear in the anonymous *Theologumena Arithmeticae*, ed. Ast, 1817.

<sup>3</sup> Theon has already quoted the oath given by Sextus in p. 94, ll. 6-7, a passage which contains probably more of the material found in *Adv. math.*, IV, 2 ff. For other citations of this oath cf. Hiller, *ad loc.* Delatte, *op. cit.*, p. 250, note. On the evident textual difficulties at this point see Hiller, *ad loc.*

Sextus:

συγκείμενος ι' ἀριθμός· α' γάρ καὶ β' καὶ γ' καὶ δ' ι' γίνεται· ὃς ἐστι τελείωτας ἀριθμός, ἐπείπερ ἐπ', αὐτὸν φθάσαντες πάλιν ἀναλύοντες ἐπὶ τὴν μονάδα καὶ ἐξ ὑπαρχῆς ποιούμεθα τὰς ἀριθμήσεις. πηγὴν κτλ. . . . εἰρήκασιν αὐτὸν διὰ τὸ καὶ αὐτοὺς ἐν αὐτῷ τὸν λόγον τῶν ἀπάντων κεῖσθαι συστάσεως, οἷον εἴθεως τοῦ τε σώματος καὶ τῆς ψυχῆς . . .

Theon:

ὑποστρεφόντων ἐπὶ μονάδα καὶ διπλά καὶ τοὺς ἔξης· τὴν δὲ δεκάδα ἐπὶ τετράδα συνίστασθαι· ἐν γάρ καὶ β' καὶ γ' καὶ δ' ἐστι ι', ὥστε τοὺς δινατωτάτους ἀριθμοὺς ἐντὸς τῆς τετράδος θεωρεῖσθαι.

Anatolius:

στρεφόντων ἐπὶ μονάδα μετὰ τὸ συμπληρούσθαι πάσαν δεκάδα· ἀλλὰ καὶ οὗτι ἐπὶ τετράδος συνίσταται η δεκάς εἰς τὰ μάλιστα τὴν τετρακτὺν ἐτίμωρ.

That all three of these passages occur in different contexts is the reason why there is divergency among them. No two contain exactly the same material, yet by pairs they show close agreement, and the same thought is implicit in all three. The close similarity of the latter parts of Anatolius and Theon leaves no doubt that both go back to the same ultimate source,<sup>1</sup> and their differences are to be ascribed to the position of the passage in Theon, as a link between the discussion of the tetraktyes and that of the first decade, and to Anatolius' tendency to condense, noticeable throughout his whole treatise. He has here left out all mention of the Pythagorean oath and the other saying,<sup>2</sup> but his own last words, "they honored the tetraktyis especially,"<sup>3</sup> its inclusion in both Sextus and Theon, and the fact that his statements about the Decad have point only as an explanation of the oath by the tetraktyis, make it hardly doubtful that this is his own arbitrary omission. As for Sextus, his passage contains all the thought of the other two—the two Pythagorean sayings: the notion that 10 is made up of 1, 2, 3, and 4, the tetraktyis;

<sup>1</sup> If there were any doubt, the verbal correspondence of Theon and Anatolius in the other passages dealing with numbers would dispel it.

<sup>2</sup> This, however, is due only to the desire to condense, for the saying is found in a collection of excerpts from Anatolius, in this form: ὅτι τὴν ἀριθμητικὴν οὐ μόνον ἑτίμα Πινθαγόρας, ἀλλὰ καὶ οἱ τοῦτο γνώριμοι ἐπιλέγοντες Ἀριθμῷ δὲ τε πάντ' ἐπέσκεψεν (see F. Hultsch, *Heronis Alexandrini Geometricorum et stereometricorum reliquiae*, Berlin, 1864, p. 279).

<sup>3</sup> This phrase, and the concluding phrase of Theon, should be compared with the end of the passage of Sextus, which is quoted below, p. 318.

that 10 includes all the numbers (he says that it is "most perfect"), because we find no new ones in counting above it. The order of the last two is reversed in Anatolius and Theon as compared with this, and in what follows the part quoted. Sextus elaborates further the powers of the numbers of the tetrakty. In general, we may conclude that we have here two versions of the same thing, a longer one represented by Sextus, and a condensed one in Theon and Anatolius, the latter having still further condensed on his own account.<sup>1</sup> Possibly more material that originally belonged in this context and is still seen there in Sextus is to be found in Theon's foregoing discussion of the tetraktyes.<sup>2</sup> To make the relationship even more probable, we shall see later that in another section Anatolius agrees word for word with Sextus in the latter's continuation of the citation above.

As to the original nature of the passage, the clue, I believe, is given by Theon and Anatolius. In each it appears as the introduction to a discussion of the numbers of the first decade; in Theon this is not so apparent, since it also happens to conclude the section about the tetraktyes, but Anatolius' employment of it should be taken as the proper one. This conclusion will recommend itself if one simply reads over the Theonian context and notes the plain evidence of its disjointedness, and furthermore that in citing these words Theon is led to make an awkward repetition of statements already introduced relative to the first tetrakty.<sup>3</sup> We have, then, in this passage the introduction to a Pythagorean treatise on the first ten numbers, and surely nothing could be more appropriate for this purpose, from the Pythagorean standpoint, than a forceful exposition of the general powers of number in the cosmos, as epitomized in 10 and the tetrakty.

<sup>1</sup> There are other versions of the material presented in the three passages quoted. The original arithmological document, of which I assume these to cite the preface, apparently repeated most of the statements later, in the chapters devoted to 4 and 10, and from these probably are derived, as far as any connection exists, the reports of Philo *De mundi opificio* cc. 15–16, Lydus *De mensibus* iii. 4, iv, 64, Wünsch, Chalcidius in *Tim.* c. 35, Hierocles in *carm. aur.* c. 20, Martianus Capella, Favonius, etc.

<sup>2</sup> See p. 312, n. 3, above.

<sup>3</sup> Cf. Theon, p. 99, ll. 21–23, with p. 93, ll. 19 ff., Hiller.

In order to clarify the situation still further, the words of Sextus immediately after what has been quoted may be compared with the following:

Sextus *Adv. math.* iv. 4  
(p. 722, 15 Bekker.):

ἡ μὲν οὖν μονὰς ἀρχή τις ὑποκείται τῆς τῶν ἄλλων ἀριθμῶν ἀπεργαστικὴ συστάσεως, ἡ δὲ δύνας μῆκους ἔστι ἀπεργαστική. καθάπερ γάρ ἐπὶ τῶν γεωμετρικῶν ἀρχῶν ὑπεδείξαμεν<sup>1</sup> πρῶτον τις ἔστιν ἡ στιγμή, εἴτα μετ' αὐτῆν ἡ γραμμὴ μῆκος ἀπλατές τυγχάνοντα, τὸν αὐτὸν τρόπον καὶ ἐπὶ τοῦ παρόντος ἡ μὲν μονὰς τὸν τῆς στιγμῆς ἐπέχει λόγον, ἡ δὲ δύνας τὸν τῆς γραμμῆς καὶ τὸν μῆκον· ποθὲν γάρ πον ἐφέρετο δυνῆς, καὶ προστιθεμένης τῇ κατὰ μῆκος διαστάσῃ τῆς κατὰ πλάτος διαστάσεως ἐπιφάνεια νοεῖται, ἀλλὰ κἄν ἐπιθεωρήσῃ τις τῇ τριάδι τετάρτην μονάδα, τοιτέστιν τέταρτον σημείον, γίνεται πυραμίς, στερεὸν σῶμα καὶ σχῆμα· καὶ γάρ μῆκος ἔχει καὶ πλάτος καὶ βάθος· ὅστε ἐν τῷ τέσσαρα ἀριθμῷ τὸν σῶματος περιέχεσθαι λογον.

Philo *De mundi opificio*  
16:

πρώτη γάρ αὕτη τὴν τοῦ στερεοῦ φύσιν ἔδειξε, τῶν πρὸ αὐτῆς ἀριθμῶν τοῖς ἀσωμάτοις ἀνακειμένων. κατὰ μὲν γάρ τὸ ἐν τάττεται τὸ λεγόμενον ἐν γεωμετρίᾳ εἰναι σημεῖον, κατὰ δὲ τὰ δύο γραμμὴν, διότι φύσις ἐκ σημείου συνίσταται. ἡ γραμμὴ δὲ ἔστι μῆκος ἀπλατές· πλάτους δὲ προσγενούένου γίνεται ἐπιφάνεια, ἡ τέτακται κατὰ τριάδα· ἐπιφάνεια δὲ πρὸς τὴν στερεοῦ φύσιν ἐνὸς δεῖται τοῦ βάθους, διπροστεθὲν τῇ τριάδι γίνεται τετράς. . . . δὲ μὴ συνιεῖς τὸ λεγόμενον ἐκ τινος παιδίας εἰσεται πάνυ συνήθους. οἱ καρνατίζοντες εἰώθασι τρία ἐν ἐπιπέδῳ προτιθέντες κάρυα ἐπιφέρειν ἐν, σχῆμα πυραμοειδὲς ἀπογεννῶντες. τὸ μὲν οὖν ἐν ἐπιπέδῳ τρίγωνον ἴσταται μέχρι τριάδος, τὸ δὲ ἐπιπέθεν τετράδα μὲν ἐν ἀριθμοῖς, ἐν δὲ σχήμασι πυραμίδα γεννᾶ, στερεόν ηδη σῶμα.

Anatolius, p. 32, 3 Hei-  
berg:

πρῶτος ἔδειξε τὴν στερεοῦ φύσιν.

σημεῖον γάρ,  
εἴτα γραμμή,

εἴτα ἐπιφάνεια,

εἴτα στερεόν, δέ ἔστι σῶμα.

τοῦτο τῶν καρνατιζόντων παιδιά, ποιούσα σχῆμα πυραμίδος.

The passage of Anatolius comes, not from the introduction, but from the chapter on the tetrad, and there is a close parallel to it in Johannes Laurentius Lydus *De mensibus* iv. 64 (Wünsch).<sup>2</sup> There can be no doubt that Philo, Anatolius, and Lydus ultimately go

<sup>1</sup> This is probably a reference to *Adv. math.* iii. 19 ff.

<sup>2</sup> πρῶτος οὖν τετράγωνος ἀριθμὸς οὗτος καὶ τετρακτύς, ἀλλὰ μὴν καὶ πρῶτος ἔδειξε τὴν στερεάν φύσιν· σημεῖον γάρ, εἴτα γραμμή, εἴτα ἐπιφάνεια, εἴτα στερεόν, δέ ἔστι σῶμα. Theon, p. 101, 11 Hiller has so abbreviated his notice of the number 4 that little can be made of the parallel passage: ἡ δὲ τετράς στερεοῦ ἔστιν εἰκών, πρῶτος τε ἀριθμὸς [καὶ] τετράγωνός ἔστιν ἐν ἀρτίοις, κτλ.

back to an identical source; the former, however, apparently used it in an unabridged form, while Anatolius and Lydus both drew from an epitomized version. Without delaying to speculate upon these matters, in support of which further arguments could be stated, let us note that, although Sextus treats of similar topics, the form of his statement is very different from that of the other three. This is probably because he drew from the introduction, the others from the body, of the common source.

Comparing the immediately succeeding words of Sextus, again, with others of the arithmological writers, a somewhat different situation appears:

*Sextus Adv. math.*  
iv. 6 ff. (Bekker, p. 723):

καὶ μὴν καὶ τὸν τῆς ψυχῆς ὡς γάρ τὸν δλον κόσμον κατὰ ἀρμονίαν λέγοντο διοικεῖσθαι, οὕτω καὶ τὸ ξῶν ψυχοῦσθαι. δοκεῖ δὲ η τέλειος ἀρμονία ἐν τρισὶ συμφωνίαις λαμβάνει τὴν ὑπόστασιν, τῇ τε διὰ τεττάρων καὶ τῇ διὰ πέντε καὶ τῇ διὰ πασῶν. η μὲν οὖν διὰ τεσσάρων ἐν ἐπιτρίψι τετταὶ λόγων, η δὲ διὰ πέντε ἐν ἡμιολίψ, η δὲ διὰ πασῶν ἐν διπλασίαι. ἐπιτρίπτος<sup>2</sup> δὲ λέγεται ἀριθμὸς δ ἔξ δλον

Anatolius, p. 32,  
15 Heiberg:

οὐ μόνον δὲ τὸν τοῦ σώματος ἐπέχει λόγον ἐν ἀριθμῷ τεττράς,<sup>1</sup> ἀλλὰ καὶ τὸν τῆς ψυχῆς ὡς γάρ τὸν δλον κόσμον φασὶ κατὰ ἀρμονίαν διοικεῖσθαι, οὕτως καὶ τὸ ξῶν ψυχοῦσθαι. δοκεῖ δὲ τέλειος ἀρμονία ἐν τρισὶ συμφωνίαις ὑφεστάναι, τῇ διὰ δ', ητις ἐν ἐπιτρίψι κεῖται λόγων, τῇ διὰ ε' ἐν ἡμιολίψ, τῇ διὰ πασῶν ἐν διπλασίαι.

Philo, *De mundi opificio* 15:

περιέχει δὲ η τετράς καὶ τὸν λόγον τῶν κατὰ μονοτικὴν συμφωνῶν, τῆς τε διὰ τεττάρων καὶ διὰ πέντε καὶ διὰ πασῶν καὶ προσέπτι δὶς διὰ πασῶν, έξ ὧν σύντημα τὸ τελειότατον πέπογεννάται. τῆς μὲν γάρ διὰ τεττάρων ὁ λόγος ἐπιτρίπτος, τῆς δὲ διὰ πέντε ἡμιόλιος, διπλάσιος δὲ τῆς διὰ πασῶν, τετραπλάσιος δὲ τῆς δὶς διὰ πασῶν. οὐδὲς ἀπαντας η τετράς ξχει περιλαβούσα τὸν μὲν γοῦν ἐπιτρίπτον ἐν τῷ τέσσαρα

symphoniae quoqueratio ex eorum numerorum qui decimanum numerum complent quasi quodam fonte demanat: siquidem ex his epitriti et sescuplares et duplices et triplices et quadruplices numeri sonique nascuntur. epitriti quidem ut quattuor adversum tres. habent enim totum numerum trientem et eius tertiam partem, id est

<sup>1</sup> Note that this parallels the final words of Sextus in the last passage quoted above. It is a very significant parallel.

<sup>2</sup> The following section, containing matter explaining certain arithmetical terms, since it is not paralleled by Anatolius, might be regarded as an addition by Sextus; on the other hand it could as easily be an instance of an omission by Anatolius for the sake of brevity. Chalcidius presents similar details, without, however, the distinctive marks of close relationship present in Sextus and Anatolius.

Sextus:

τινὸς ἀριθμοῦ συνεστηκώς καὶ ἐκ τοῦ τρίτου μέρους ἐκείνου, ὡς ἔχει δὲ δικτὼ πρὸς τὸν ἔξι καὶ γάρ αὐτὸν τὸν ἔξι περισσήκε καὶ τὸ τρίτον αὐτοῦ, τοντέστι τὴν δύναδα. ἡμιόλιος δὲ καλεῖται, δταν περιέχῃ ἀριθμὸς ἀριθμὸν καὶ τὸ ἡμισυ ἐκείνου, ὡς ἔχει δὲ ἑννέα πρὸς τὸν ἔξι. συνέστηκε γάρ ἐκ τοῦ ἔξι καὶ ἐκ τοῦ ἡμισεος αὐτοῦ, τοντέστι τῶν τριῶν. διπλασίων δὲ προσαγορεύεται δὲ δυσὶν ἀριθμοῖς ἵσος, ὡς δὲ τέσσαρα πρὸς τὸν δύο· δις γάρ τὸν αὐτὸν περιέσχηκεν. ἀλλὰ γάρ τοντων οὕτως ἔχοτων, καὶ κατὰ τὴν ἀρχῆθεν ὑπόθεσιν τεσσάρων δυντων ἀριθμῶν, τοῦ τε ἑνὸς καὶ δύο καὶ τρία καὶ τέσσαρα, ἐν οἷς ἐλέγομεν καὶ τὴν τῆς ψυχῆς ἰδέαν περιέχεσθαι κατὰ τὸν ἑναρμόνιον λόγον, δὲ μὲν δ' τοῦ β' καὶ δὲ β' τοῦ α' διπλασίος, ἐν φ' κεῖται δὲ διὰ πασῶν συμφωνία, δὲ γ' τοῦ β' ἡμιόλιος, περιέχων αὐτὸν καὶ τὸ ἡμισυ, τὴν διὰ γάρ αὐτὸν τὸν δύο

Anatolius:

δυντων δὲ ἀριθμῶν τεττάρων τῶν πρώτων α' β' γ' δ', ἐν τούτοις καὶ δὲ τῆς ψυχῆς ἰδέα περιέχεται κατὰ τὸν ἑναρμόνιον λόγον, δὲ μὲν δ' τοῦ β' καὶ δὲ β' τοῦ α' διπλασίος, ἐν φ' κεῖται δὲ διὰ πασῶν συμφωνία, δὲ γ' τοῦ β' ἡμιόλιος, περιέχων αὐτὸν καὶ τὸ ἡμισυ, τὴν διὰ πέντε συμφωνίαν ὑποβάλλει, δὲ δ'

Philo:

πρὸς τρία, τὸν δὲ ἡμιόλιον ἐν τῷ τρίᾳ πρὸς δύο, τὸν δὲ διπλάσιον ἐν τῷ δύο πρὸς ἦν ἡ τέσσαρα πρὸς δύο, τὸν δὲ τετραπλάσιον ἐν τῷ τέσσαρα πρὸς δύο.

Chalcidius:

unum. sesquiplares uero, ut tres aduersum duo; habent enim tres totum numerum dualem et eius partem dimidiad, id est unum. duplices uero ut sunt duo aduersus unum. triplices porro ut sunt tres aduersus unum. et quadruplices ut sunt quattuor adversus unum. epitritus autem in calculando idem est qui diatesaron dicitur in canendo. sesquiplaris uero idem est qui diapente dicitur in canendo. duplex uero qui diapason dicitur in canendo,<sup>1</sup> quadruplex qui diadiapason dicitur in canendo.

<sup>1</sup> At this point Wrobel would add "triplex qui diapason et diapente dicitur in canendo," following Fabricius and thinking that Chalcidius omitted the words, which appear neither in the MSS nor the first edition, through error or forgetfulness. Note, however, that the passage agrees with Theon, p. 58, 13 ff., as it stands.

Sextus:

περιέσχηκε καὶ τὸ  
ῆμισυ τούτου, ὅθεν  
καὶ τὴν διὰ πέντε  
συμφωνίαν ὑπέβαλ-  
λεν), ὃ δὲ τέσσαρα  
τοῦ τρία ἐπίτριτος,  
ὑπέκειτο δὲ καὶ ἐν  
τούτῳ ἡ διὰ τεσ-  
σάρων συμφωνία.  
ὅστε εἰκότως τὸν  
τέσσαρα ἀριθμὸν  
παρὰ τοῖς Πιθαγο-  
ρικοῦς εἰρῆσθαι πη-  
γήν ἀεράου φύσεως  
διέδωματ' ἔχονταν.

Anatolius:

τοῦ γ' ἐπίτριτος, ἐν  
φὴ διὰ δ' συμφωνία.  
εἰ δὲ ἐν τῷ δ' ἀριθμῷ  
τὸ πᾶν κεῖται ἐκ  
ψυχῆς καὶ σώματος,  
ἀληθὲς ἄρα καὶ ὅτι  
αἱ συμφωνίαι πᾶσαι  
κατ' αὐτὸν τελοῦν-  
ται.<sup>1</sup>

Philo:

Chalcidius:

In Theon's arithmology, p. 101, 12 Hiller, he merely says *καὶ αἱ συμφωνίαι δὲ πᾶσαι κατ' αὐτὸν συμπτληροῦνται, ὡς ἐδείχθη*. This reference leads us back to p. 93, 17: *ἐπειδὴ πάντες οἱ τῶν συμφωνιῶν εὑρέθησαν λόγοι, καθὰ δέδεικται, ἐν τῇ τῆς δεκάδος τετρακτύν, καὶ περὶ τούτων πρότερον λεκτέον. τὴν μὲν γὰρ τετρακτύν συνέστησεν ἡ δεκάς. ἐν μὲν γὰρ καὶ β' καὶ γ' καὶ δ' ι' α' β' γ' δ'. ἐν δὲ τούτοις τοῖς ἀριθμοῖς ἔστιν ἡ τε διὰ τεσσάρων συμφωνία ἐν ἐπιτρίτῳ λόγῳ καὶ ἡ διὰ πέντε ἐν ἡμιολίῳ καὶ ἡ διὰ πασῶν ἐν διπλασίᾳ καὶ ἡ δὶς διὰ πασῶν ἐν τετραπλασίᾳ· ἔξ ὧν συμπληροῦται τὸ ἀμετάβολον διάγραμμα.* And in turn the reference here is to p. 58, 13: *πάσας δὲ τὰς συμφωνίας περιέχει ἡ τετρακτύς. συνέστησε μὲν γὰρ αὐτὴν α' καὶ β' καὶ γ' καὶ δ'. ἐν δὲ τούτοις τοῖς ἀριθμοῖς ἔστιν ἡ τε διὰ τεσσάρων συμφωνία καὶ ἡ διὰ πέντε καὶ ἡ διὰ πασῶν, καὶ ὁ ἐπίτριτος λόγος καὶ ἡμιόλιος καὶ διπλάσιος καὶ τριπλάσιος καὶ τετραπλάσιος.*

Among these passages, there certainly exists an agreement between Sextus and Anatolius which is indicative of a common source; the others do not share this verbal similarity, and, besides, they include among the harmonic ratios the diapason, making four, while Sextus and Anatolius report only three. Chalcidius and Theon, here as elsewhere,<sup>2</sup> show marks of close relationship; they

<sup>1</sup> The concluding sentence of Anatolius is obviously modeled after what is seen in Sextus, but modified to suit its new context.

<sup>2</sup> This alliance is manifested by several significant minor identities of phrase; for instance, of the triad, Theon says *πρώτη λέγεται πάντα εἶναι* and Chalcidius

both mention the triple ratio, and each regards the number four as the tetraktys. The conditions may be explained by concluding that Sextus and Anatolius for this passage depend on the introduction of the arithmological treatise, Philo, Theon, and Chalcidius upon its chapter on 4. The latter two, however, are probably derived from some common ancestor between the ultimate common source and themselves; this will explain their slight disagreement with Philo. The likeness between Sextus and Anatolius, however, is a most important confirmation of the conclusion reached above with regard to the first set of passages cited.

Thus the passage of Sextus is seen to be connected most intimately with a lost book, the existence of which, however, is as certain as that of the *Iliad*—namely, the common source of Anatolius, Theon, Philo, and Lydus, which determines the form of the first two throughout the whole length of their arithmologies,<sup>1</sup> and from which Philo and Lydus excerpted lengthy fragments identifiable by their close likeness to Theon and Anatolius.<sup>2</sup> Besides these, Chalcidius<sup>3</sup> was certainly affected by it, probably also the *Theologumena Arithmeticae*,<sup>4</sup> Varro,<sup>5</sup> Hermippus of Berytus,<sup>6</sup> and indirectly Macrobius,

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*tria . . . . dicta sunt omnia;* but Anatolius and Lydus (ii. 8) both have *ὅτι πρῶτος ῥὰ πάντα σημαντεῖ*. Otherwise the passages are much alike. Again, in the chapter on 7, Chalcidius inserts a block of material much resembling a similar block inserted by Theon, which disagrees with the corresponding parts of Anatolius, Philo, and Lydus.

<sup>1</sup> It is to be identified with the “grand traité d'époque alexandrine” mentioned by Delatte, *op. cit.*, pp. 140, 207.

<sup>2</sup> See Philo *De mundi opificio* cc. 3, 15–16, 20, 30–42; Lydus *De mensibus* ii. 7, 9, 11, 12; iii. 4; iv. 64 (all in part). The connection of other passages of both authors with this source is debatable.

<sup>3</sup> In his commentary on the *Timaeus* cc. 35–37.

<sup>4</sup> An anonymous treatise first edited by Ast in 1817, thought by most to have been compiled by Iamblichus; its chief sources are Anatolius and the *Theologumena Arithmeticae* of Nicomachus of Gerasa (this latter is otherwise known only through Photius, *Bibliotheca*, cod. 187). Probably Nicomachus' *Theologumena* also was influenced by the anonymous arithmologist.

<sup>5</sup> In the first book of the *Hebdomades* or *Imagines*, ap. Aulus Gellius *Noct. Att.* iii. 10, and in *Tubero* or *De origine humana*, ap. Censorinus *De die natali* 9, 1 ff.

<sup>6</sup> Clement of Alexandria, *Strom.* VI. xvi (see especially sec. 145, 2) quotes his *περὶ ἐθνομάθος*.

Martianus Capella, Favonius, and Isidore of Seville.<sup>1</sup> By comparisons among these and other writers it would still be possible to reproduce a large part of this source in its original form. It consisted of an introduction and ten chapters dealing with the ten numbers.<sup>2</sup> The study of its transmission, which would be too lengthy to attempt here, would show that it circulated in several versions and epitomes,<sup>3</sup> and that its influence was spread still farther by those who derived their arithmology from it. But although it was the most generally quoted of all ancient arithmologies, its very name and that of its author are lost and probably will never be known. It was probably written by at least 100 B.C., for Varro seems to have been influenced by it; Philo,<sup>4</sup> in the early first century A.D., certainly was.

With regard to the question of the share of Posidonius in promulgating Pythagorean lore, the conclusion must be that even if the whole of Sextus *Adv. math.* vii. 91–109 is his, the Pythagorean part of it must nevertheless be a citation on his part from the introduction of an already existent work, which was known to Sextus also from another source, that used in iv. 2 ff. This treatise, and not Posidonius, was the ultimate source of information used by the long list of writers just named.

In conclusion, let us consider briefly the question whether or not the citation of vii. 91 ff. belongs to Posidonius, reviewing the arguments that connect him with the arithmological traditions. First, it has been claimed<sup>5</sup> that the mention of Posidonius' name at

<sup>1</sup> The sources of these writers are not so easily determined. Macrobius in his commentary on the *Somnium Scipionis* (i. 6) parallels extensive passages of the *Theologumena Arithmeticae*.

<sup>2</sup> There is a possibility that it formed a part of a large Pythagorean work dealing with the mathematical sciences generally.

<sup>3</sup> Anatolius, Theon, and others seem to have used epitomes; Philo, discussing the number 7 in *De mundi opificio* 30–42, and Sextus, quoting the Introduction, apparently had access to the unabridged work. As has been seen above, Anatolius also quoted a section of this Introduction.

<sup>4</sup> Philo was the author of a book, now lost, called *περὶ ἀριθμῶν*, mentioned by title in *Vit. Mos.* iii. 11, *Quaestiones et solutiones in Genesim* iv. 110, 151, and less definitely in *De mundi opificio* cc. 16 and 43, *Quaest. et sol. in Gen.* ii. 14 and iii. 49. Apparently he collected here all that is said of the numbers in his extant works, and the treatise was doubtless affected, as they are, by the anonymous arithmologist.

<sup>5</sup> By Schmekel, *loc. cit.*

the beginning marks him as the author. This is a double-edged argument and by no means conclusive; for ancient writers quite as often concealed the name of their primary source as they revealed it, and frequently they veiled their action by mentioning freely the authorities named in their real source.<sup>1</sup> Furthermore, we have now seen that in any case Posidonius himself must have been quoting; and this will also apply to the purely subjective argument that attempts to decide, from the nature of the passage, that the whole belongs to him.<sup>2</sup>

More satisfactory in some respects is an argument based on the commentary of Chalcidius, c. 50, where the following occurs:

ult igitur animam sensibilis mundi tamquam permissa usurpandi licentia nasci, cognitricem tamen rerum omnium, quae sunt tam intellegibles quam sensibiles. est porro Pythagoricum dogma similia nonnisi a similibus suis comprehendi. quod etiam Empedocles sequens ait in suis uersibus:

terram terreno comprehendimus, aethera flammis,  
humorem liquido, nostro spirabile flatu,  
pacem tranquillo, litem quoque litigioso.

haec quippe constituebat elementa et initia uniuersitatis, ex quibus animae quoque censebat constare substantiam.

The thought of this passage is parallel to Sextus *Adv. math.* vii. 91 ff., and the citation of the same verses of Empedocles, and the possible reference to Philolaus, are remarkable. The reappearance of all these matters in conjunction could be offered as evidence that the passage in Sextus is a unity, and from the pen of Posidonius; it has, in fact, been argued that Chalcidius (probably through Adrastus) and Sextus both depend upon him.<sup>3</sup> Even if this is so, however, it does not prove that the Posidonian quotation in Sextus extends over the arithmological sections as well, of which in any case he could not have been the author.

<sup>1</sup> Cf. Eysenhardt's introduction to his edition of Martianus Capella, p. xxxii, and Hiller in *Rheinisches Museum*, XXVI (1871), pp. 582 ff., cited by Switalski, *op. cit.*, p. 62.

<sup>2</sup> For the controversy over the exact extent of the Posidonian quotation, cf. Zeller and Schmekel, cited above, p. 310, n. 2, and R. M. Jones, *The Platonism of Plutarch*, Chicago dissertation, Menasha, 1916, p. 77, n. 21. The latter agrees with Zeller that the Pythagorean material does not belong to Posidonius. It may be noted that at the conclusion of the debated passage Sextus (vii. 110) appends the words *ταῦτα μὲν οἱ Πυθαγορικοί*.

<sup>3</sup> Borghorst, *op. cit.*, p. 60.

Finally, there is some interest in the citation of Posidonius by Theon in his arithmology, p. 103, 16 ff.: *ἐπόμενος δὲ τῇ φύσει καὶ ὁ Πλάτων ἔξ ἐπτὰ ἀριθμῶν συνέστησε τὴν ψυχὴν ἐν τῷ Τιμαιῷ. ἡμέρα γὰρ καὶ νύξ, ὡς φησι δὲ Ποσειδώνιος, ἀρτίου καὶ περιπτοῦ φύσιν ἔχουσι. μὴν δὲ καθ' ἔβδομάδας τέσσαρας συμπληροῦται, τῇ μὲν πρώτῃ ἔβδομάδι διχοτόμου τῆς σελήνης ὀρωμένης, κτλ.* At this point, it may be noted, begins a block of material<sup>1</sup> which, unlike most of the chapter on the number 7, seems to be taken from a source different from that of Anatolius—that is, probably not from the anonymous arithmology of which we have spoken.<sup>2</sup> Opinions as to the extent of the Posidonian citation may, of course, differ; there seems to be no reason, however, to think that it did not at least include the sentence describing the influence of the number 7 on the moon and the lunar month. Nevertheless, all that can be held proven is that Posidonius was interested in Pythagorean arithmology, and that he may have influenced some lines of its tradition, not that he was the author of any part of the general source of Theon, Anatolius, Philo, and the rest. At most, he may have been responsible for introducing alien elements into the descendants of this arithmology.

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<sup>1</sup> Especially p. 104, ll. 1–5, but in all the rest of the section about the hebdomad there are striking differences of detail between Theon and Anatolius.

<sup>2</sup> This special source, however, was probably itself influenced by the Anonymous.